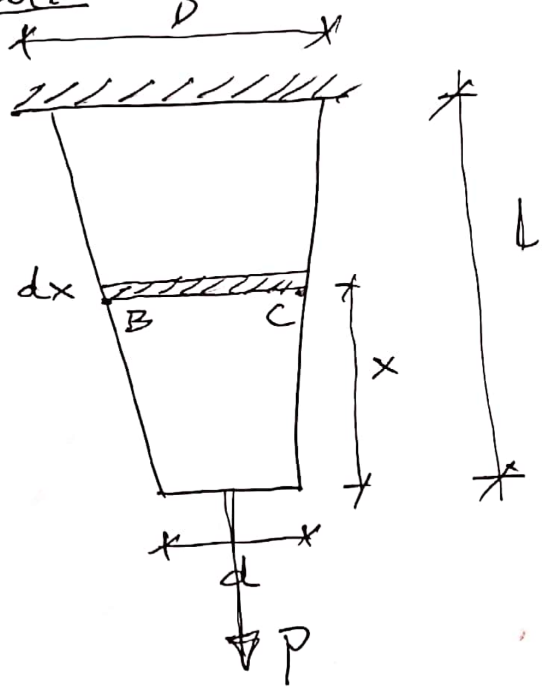
ELONGATION of TAPERING ROD

(a) Circular in X-section



Consider the diagram above as annotated. It is subjected to an axial pull of P. Therefore the stress at every point along the length of this rod will vary since the cross-sectional area vary from point to point.

Let the diameter of the rod at BC, which ~~be d_1. Then d_1 from~~ is at a distance x from ~~the~~ smaller end be d_1. Then, by proportion

$$d_1 = d + \frac{x}{L}\left(D - d\right) \qquad (1)$$

The the area of the cross-section at BC

$$A = \pi r^2$$
$$= \pi \left[\frac{d + \frac{x}{L}(D-d)}{2} \right]^2$$
$$= \frac{\pi}{4} \left[d + \frac{x}{L}(D-d) \right]^2 \quad \text{——— (2)}$$

Now, the elongation δL of the infinitesimal dx, recalling that:

$$\delta L = \frac{PL}{AE}$$

is

$$\delta L = \frac{P dx}{AE}$$

$$= \frac{P dx}{\pi \left[d + \frac{x}{L}(D-d) \right]^2 E} \quad \text{——— (3)}$$

To find the total elongation of the rod, we integrate the eqn 3 over the whole length. That is

$$\delta L_{total} = \int_0^L \frac{4P dx}{\pi \left[d + \frac{x}{L}(D-d) \right]^2 E}$$

$$= \frac{4P}{\pi E}\left(\frac{-l}{D-d}\right)\left[\frac{1}{d+(D-d)\frac{x}{l}}\right]_0^l$$

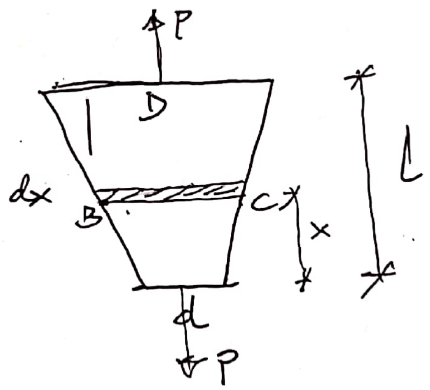
$$= \frac{4P}{\pi E}\left(\frac{-l}{D-d}\right)\left(\frac{1}{D}-\frac{1}{d}\right)$$

$=$ ⋮

$=$ ⋮

$$= \frac{4PL}{\pi E D d}$$

b) Rectangular or Square in Cross-section



Consider a rod square in cross-section, and tapered from D to d at the other end and loaded as shown. At B,

$$d_1 = d + \frac{x}{l}(D-d)$$

Then the cross-section of dx (infinitesimal)

$$A = d \times d = \left[d + \frac{x}{L}(D-d)\right]^2$$

thus elongation δl for the length dx

$$\delta l = \frac{Pl}{AE} = \frac{Pdx}{AE} = \frac{Pdx}{\left(d+[D-d]\frac{x}{l}\right)^2 E}$$

for the whole length, the total elongation

$$\delta L_{total} = \int_0^l \frac{Pdx}{\left[d+\frac{x}{l}(D-d)\right]^2 E}$$

$$= \quad \vdots$$

$$= \quad \vdots$$

$$= \frac{P}{E}\left(\frac{-l}{D-d}\right)\left[\frac{1}{d+\frac{(D-d)x}{l}}\right]\Bigg|_0^l$$

$$= \quad \vdots$$

$$= \frac{P}{E}\left[\frac{-l}{D-d}\right]\left[\frac{1}{D}-\frac{1}{d}\right]$$

$$= \frac{Pl}{EDd}$$